1.设$y=y(x)$是由方程$y^2+2\ln y=x^2$所确定的隐函数，则$\frac{dy}{dx}=$( ).

2.设函数$y=y(x)$是由方程$\ln(x^2+y)=x^3y+\sin x$确定的隐函数，则$\left.\frac{dy}{dx}\right|_{x=0}=$( ).

3.设函数$y=y(x)$由方程$e^{xy}+y^2=\cos x$所确定，则$\frac{dy}{dx}=$( ).

4.已知$y=y(x)$是由方程$xy=e^{y-x}$确定的函数，则$dy=$( ).

计算题

1.设函数$f(x)=x^{a^a}+a^{x^a}+a^{a^x}$,求$f'(x)$.

2.设函数$y=x^{\pi}+\pi^{x}+\sqrt[x]{\pi}+\sqrt[\pi]{\pi}$,求$y'$.

3.$y=2x^3\frac{\sqrt[4]{(1+x^2)(1-x)}}{\sqrt{e^x(x^3-\sqrt{x})}}$,求$y'(x)$.

4.已知函数$y=\ln\sqrt{\frac{(1-x)e^x}{\arccos x}}$,求$y'(0)$.

5.设函数$f(x)$可导，$y=f\left(\frac{2x-2}{2x+1}\right)$,且$f'(x)=\arctan\sqrt{x}$,求$\left.\frac{dy}{dx}\right|_{x=-1}$

6.已知函数$y=x^{1-x^2}+\frac{1}{1+x^2}\sqrt{\frac{(x+1)\sin x}{e^{x^2}(3-\sqrt{x})}}$,求$y'(x)$

7.已知函数$y=\frac{\sqrt{x+2}(3-x)^4}{(x+1)^5}$,求$\frac{dy}{dx}$.

8.设$y=\sqrt{x\sin x\sqrt{1-e^x}}$,求$y'$.

9.设$y=y(x)$是由方程$x^2-3xy+y^2+1=e^{xy}$确定的隐函数，求y'.

10.求由方程$x-y+\frac{1}{2}\sin y=0$所确定的隐函数y的二阶导数$\frac{d^2y}{dx^2}$.

11.设$y(x)$是$y+e^y=x+\cos x$由方程确定的隐函数，求$y'(0)$及$y''(0)$.

12.$y=f(x)$是由$x^3+y^3-\sin 3x+6y=0$确定，求$dy\big|_{x=0}$.

13.设$y=f(x)$是由$e^y+xy=0$确定，求$\frac{d^2y}{dx^2},\left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{e}}$.

14.设函数$y=y(x)$由方程$\sin(x^2+y^2)+e^x-xy^2=0$所确定，求$\frac{dy}{dx}$.

15.设$y=y(x)$是由方程$y^3=x+\arccos(xy)$所确定的函数，求$\frac{dy}{dx}$.

16.已知方程$x^2+xy+y^2=4$所确定的隐函数为$y=y(x)$,求$\frac{dy}{dx},\frac{d^2y}{dx^2}$.

17.设$y=y(x)$由方程$2y-x=(x-y)\ln(x-y)$确定的隐函数，求dy.

18.设$y=y(x)$是由方程组$\begin{cases} x=3t^2+2t+3 \\ e^y\sin t-y+1=0 \end{cases}$所确定的隐函数，求$\left.\frac{d^2y}{dx^2}\right|_{t=0}$.

19.求与曲线$y=x^3-3x^2+5$相切，与直线$x+9y-1=0$垂直的切线方程.

1.答案：$\dfrac{xy}{y^2+1}$

解：$\because y=y(x)$是由方程$y^2+2\ln y=x^2$所确定的隐函数.$\therefore \dfrac{dy}{dx}=-\dfrac{F_x}{F_y}=-\dfrac{-2x}{2y+\dfrac{2}{y}}=\dfrac{2x}{\dfrac{2y^2+2}{y}}=\dfrac{xy}{y^2+1}$.

2.答案： 1

解：$\because$函数$y=y(x)$是由方程$\ln(x^2+y)=x^3y+\sin x$确定的隐函数，$\therefore \dfrac{dy}{dx}=-\dfrac{F_x}{F_y}=-\dfrac{\dfrac{2x}{x^2+y}-3x^2y-\cos x}{\dfrac{1}{x^2+y}-x^3}$

$$-\frac{\dfrac{2x-3x^2y(x^2+y)-\cos x(x^2+y)}{x^2+y}}{\dfrac{1-x^3\left(x^2+y\right)}{x^2+y}}=-\frac{2x-3x^2y(x^2+y)-\cos x(x^2+y)}{x^2+y}\frac{x^2+y}{1-x^3\left(x^2+y\right)}=-$$

$-\dfrac{2x-3x^2y(x^2+y)-\cos x(x^2+y)}{1-x^3\left(x^2+y\right)}$.当$x=0$时$y=1.\Rightarrow \left.\dfrac{dy}{dx}\right|_{\substack{x=0,\\y=1}}=1$.

3.答案：$-\dfrac{ye^{xy}+\sin x}{xe^{xy}+2y}$

解：$\because$函数$y=y(x)$由方程$e^{xy}+y^2=\cos x$所确定，则$\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}=-\dfrac{ye^{xy}+\sin x}{xe^{xy}+2y}$

4.答案：$-\dfrac{y+e^{y-x}}{x-e^{y-x}}$

解：$\because y=y(x)$是由方程$xy=e^{y-x}$确定的函数,$\therefore \dfrac{dy}{dx}=-\dfrac{F_x}{F_y}=-\dfrac{y+e^{y-x}}{x-e^{y-x}}$

计算题

1.解：$\because$函数$f(x)=x^{a^a}+a^{x^a}+a^{a^x}$,$\therefore f'(x)=\left(x^{a^a}+a^{x^a}+a^{a^x}\right)'=\left(x^{a^a}\right)'+\left(a^{x^a}\right)'+\left(a^{a^x}\right)'=a^ax^{a^a-1}+a^{x^a}ax^{a-1}\ln a+$ $a^{a^x}a^x\ln^2 a$.

2.解：$\because$函数$y=x^\pi+\pi^x+\sqrt[\pi]{x}+\sqrt[x]{\pi}+\sqrt[\pi]{\pi}$,$\therefore y'=\left(x^\pi+\pi^x+\sqrt[x]{\pi}+\sqrt[\pi]{\pi}\right)'=\left(x^\pi\right)'+\left(\pi^x\right)'+\left(\sqrt[x]{\pi}\right)'+\left(\sqrt[\pi]{\pi}\right)'=\pi x^{\pi-1}$ $+\pi^x\ln\pi+\dfrac{1}{\pi}x^{\frac{1}{\pi}-1}-\sqrt[x]{\pi}\dfrac{1}{x^2}\ln\pi$.

3.解： $y=2x^3\dfrac{\sqrt[4]{(1+x^2)(1-x)}}{\sqrt{e^x(x^3-\sqrt{x})}}$利用对数求导法：等式两侧同时取对数ln $\Rightarrow \ln y=3\ln 2x+\dfrac{1}{4}\ln$ $(1+x^2)+\dfrac{1}{4}\ln(1-x)-\dfrac{1}{2}\ln e^x-\dfrac{1}{2}\ln(x^3-\sqrt{x})$.等式两侧同时对x求导：$\dfrac{y'}{y}=\dfrac{3}{x}+\dfrac{x}{2(1+x^2)}-\dfrac{1}{4(1-x)}-\dfrac{1}{2}-$ $\dfrac{3x^2-\dfrac{1}{2\sqrt{x}}}{2(x^3-\sqrt{x})}$.$y'=y[\dfrac{3}{x}+\dfrac{x}{2(1+x^2)}-\dfrac{1}{4(1-x)}-\dfrac{1}{2}-\dfrac{3x^2-\dfrac{1}{2\sqrt{x}}}{2(x^3-\sqrt{x})}]=2x^3\dfrac{\sqrt[4]{(1+x^2)(1-x)}}{\sqrt{e^x(x^3-\sqrt{x})}}[\dfrac{3}{x}+\dfrac{x}{2(1+x^2)}-$

$\frac{1}{4(1-x)}-\frac{1}{2}-\frac{3x^2-\frac{1}{2\sqrt{x}}}{2(x^3-\sqrt{x})}]$

4.解：$y=\ln\sqrt{\frac{(1-x)e^x}{\arccos x}}=\frac{1}{2}\ln(1-x)+\frac{1}{2}\ln e^x-\frac{1}{2}\ln\arccos x.\Rightarrow y'=-\frac{1}{2(1-x)}+\frac{1}{2}+\frac{1}{2\arccos x}\frac{1}{\sqrt{1-x^2}}$,

$y'(0)=\frac{1}{\pi}$.

5.解：$\because$函数$f(x)$可导，$y=f\left(\frac{2x-2}{2x+1}\right), f'(x)=\arctan\sqrt{x}.\therefore\frac{dy}{dx}=f'\left(\frac{2x-2}{2x+1}\right)\left(\frac{2x-2}{2x+1}\right)'=\arctan\left(\sqrt{\frac{2x-2}{2x+1}}\right)$

$\times\frac{2(2x+1)-2(2x-2)}{(2x+1)^2},\left.\frac{dy}{dx}\right|_{x=-1}=6\arctan 2.$

6.解：对等式$y=x^{1-x^2}+\frac{1}{1+x^2}\sqrt{\frac{(x+1)\sin x}{e^{x^2}(3-\sqrt{x})}}$进行拆分：$y=y_1+y_2\Rightarrow y_1=x^{1-x^2}, y_2=\frac{1}{1+x^2}\sqrt{\frac{(x+1)\sin x}{e^{x^2}(3-\sqrt{x})}}$

对等式y_1,y_2两侧同时取对数ln：$\ln y_1=(1-x^2)\ln x;\ln y_2=-\ln(1+x^2)+\frac{1}{2}\ln(1+x)+\frac{1}{2}\ln\sin x-\frac{1}{2}x^2-$

$\frac{1}{2}\ln(3-\sqrt{x})$;对等式$\ln y_1=(1-x^2)\ln x$两侧同时对x求导：$\frac{y_1'}{y_1}=-2x\ln x+\frac{1-x^2}{x}$.对等式$\ln y_2=$

$-\ln(1+x^2)+\frac{1}{2}\ln(1+x)+\frac{1}{2}\ln\sin x-\frac{1}{2}x^2-\frac{1}{2}\ln(3-\sqrt{x})$两侧同时对x求导：$\frac{y_2'}{y_2}=-\frac{2x}{1+x^2}+\frac{1}{2(1+x)}$

$+\frac{\cos x}{2\sin x}-x+\frac{1}{2(3-\sqrt{x})}\frac{1}{2\sqrt{x}};\therefore y'(x)=y_1'+y_2'=x^{1-x^2}\left(-2x\ln x+\frac{1-x^2}{x}\right)+\frac{1}{1+x^2}\sqrt{\frac{(x+1)\sin x}{e^{x^2}(3-\sqrt{x})}}(-\frac{2x}{1+x^2}$

$+\frac{1}{2(1+x)}+\frac{\cos x}{2\sin x}-x+\frac{1}{2(3-\sqrt{x})}\frac{1}{2\sqrt{x}})$.

7.解：等式两侧同时取对数ln $\Rightarrow\ln y=\frac{1}{2}\ln(x+2)+4\ln(3-x)-5\ln(x+1)$.等式两侧同时对x求导

$\frac{y'}{y}=\frac{1}{2(x+2)}-\frac{4}{(3-x)}-\frac{5}{(x+1)}.\Rightarrow y'=y[\frac{1}{2(x+2)}-\frac{4}{(3-x)}-\frac{5}{(x+1)}]\Rightarrow\frac{\sqrt{x+2}(3-x)^4}{(x+1)^5}[\frac{1}{2(x+2)}-$

$\frac{4}{(3-x)}-\frac{5}{(x+1)}]$.

8.解：$y=\sqrt{x\sin x\sqrt{1-e^x}}$.等式两侧同时取对数ln：$\ln y=\frac{1}{2}\ln x\sin x+\frac{1}{4}\ln(1-e^x)$.$\Rightarrow$等式两侧

同时对x求导：$\frac{y'}{y}=\frac{\sin x+x\cos x}{2x\sin x}+\frac{-e^x}{4(1-e^x)}\Rightarrow y'=y[\frac{\sin x+x\cos x}{2x\sin x}+\frac{-e^x}{4(1-e^x)}]=\sqrt{x\sin x\sqrt{1-e^x}}$

$[\frac{\sin x+x\cos x}{2x\sin x}+\frac{-e^x}{4(1-e^x)}]$.

9.解：$\because$ $y=y(x)$是由方程$x^2-3xy+y^2+1=e^{xy}$确定的隐函数，$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{2x-3y-ye^{xy}}{-3x+2y-xe^{xy}}$

10.解：$\because$ $x-y+\frac{1}{2}\sin y=0.\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{1}{-1+\frac{1}{2}\cos y}=\frac{2}{2-\cos y}.\frac{d^2y}{dx^2}=\frac{-2y'\sin y}{(2-\cos y)^2}=\frac{-4\sin y}{(2-\cos y)^3}$

11.解：$\because$ $y(x)$是$y+e^y=x+\cos x$由方程确定的隐函数.$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{-1+\sin x}{1+e^y}$.当$x=0$时，$y=0$.

$$y'(0)=\frac{1}{2}.\frac{d^2y}{dx^2}=\frac{-(1+e^y)\cos x-y'e^y(1-\sin x)}{(1+e^y)^2}=\frac{\frac{-(1+e^y)^2\cos x-e^y(1-\sin x)^2}{1+e^y}}{(1+e^y)^2}=\frac{-(1+e^y)^2\cos x-e^y(1-\sin x)^2}{(1+e^y)^3}$$

将$x=0$，$y=0$代入原式：$y''(0)=-\frac{5}{8}$.

12.解：$\because$ $y=f(x)$是由$x^3+y^3-\sin 3x+6y=0$确定,$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{3x^2-3\cos 3x}{3y^2+6}$.当$x=0$时，$y=0$

$.dy|_{x=0}=\frac{1}{2}dx.$

13.解：$\because$ $y=f(x)$是由$e^y+xy=0$确定,$\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{y}{e^y+x}.\frac{d^2y}{dx^2}=\frac{-y'(e^y+x)-yy'e^y+y}{(e^y+x)^2}$.将$x=\frac{1}{e}$代入

原式得$y=-1$.将其带入$\frac{d^2y}{dx^2}\Rightarrow -\frac{5}{8}e^2$.

14.解：$\because$ $y=y(x)$由方程$\sin(x^2+y^2)+e^x-xy^2=0$所确定.$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{2x\cos(x^2+y^2)+e^x-y^2}{2y\cos(x^2+y^2)-2xy}$.

15.解：$\because$ $y=y(x)$是由方程$y^3=x+\arccos(xy)$所确定的函数.$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{-1+\frac{y}{\sqrt{1-(xy)^2}}}{3y^2+\frac{x}{\sqrt{1-(xy)^2}}}\Rightarrow$

$$\frac{\frac{\sqrt{1-(xy)^2}-y}{\sqrt{1-(xy)^2}}}{\frac{3y^2\sqrt{1-(xy)^2}+x}{\sqrt{1-(xy)^2}}}=\frac{\sqrt{1-(xy)^2}-y}{3y^2\sqrt{1-(xy)^2}+x}.$$

16.解： 方程$x^2+xy+y^2=4$所确定的隐函数为$y=y(x)$.$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{2x+y}{x+2y}.\frac{d^2y}{dx^2}=\left(-\frac{2x+y}{x+2y}\right)'$

$=\frac{(-2-y')(x+2y)-(1+2y')(-2x-y)}{(x+2y)^2}$.

17.解：$\because$ $y=y(x)$由方程$2y-x=(x-y)\ln(x-y)$确定的隐函数.$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{-1-\ln(x-y)-1}{2+\ln(x-y)+1}$

$=\frac{2+\ln(x-y)}{3+\ln(x-y)}.\therefore dy=\frac{2+\ln(x-y)}{3+\ln(x-y)}dx.$

18.解：∵ $\begin{cases} x=3t^2+2t+3 \\ e^y\sin t-y+1=0 \end{cases}$ 所确定的隐函数，则由 $x=3t^2+2t+3$ 得 $\dfrac{dx}{dt}=6t+2.$ 由 $e^y\sin t-y+1=0$

得 $y|_{t=0}=1.$ 及 $\dfrac{dy}{dt}=\dfrac{e^y\cos t}{1-e^y\sin t}=\dfrac{e^y\cos t}{2-y}.\therefore\left.\dfrac{dy}{dt}\right|_{t=0}=e.\dfrac{dy}{dx}=\dfrac{e^y\cos t}{(2-y)(6t+2)}.$ 故 $\dfrac{d^2y}{dx^2}=\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)\dfrac{dt}{dx}=$

$$\frac{\left(\dfrac{dy}{dt}e^y\cos t-e^y\sin t\right)(2-y)(6t+2)-e^y\cos t\left[6(2-y)-\dfrac{dy}{dt}(6t+2)\right]}{(2-y)^2(6t+2)^3}.\left.\frac{d^2y}{dx^2}\right|_{t=0}=\frac{2e^2-3e}{4}.$$

19.解：∵曲线 $y=x^3-3x^2+5$ 的切线与直线 $x+9y-1=0$ 垂直即 $y'=3x^2-6x\perp x+9y-1=0\Rightarrow k_{切}=9.$ 将 $k_{切}=9$ 代入 $y'=3x^2-6x\Rightarrow$ 解得 $x_1=3,x_2=-1$ 代入方程得 $y_1=5,y_2=1.$ 在 $(3,5),(-1,1)$ 处的切线方程为 $y-5=9(x-3).\Rightarrow y=9x-22.\ \ y-1=9(x+1)\Rightarrow y=9x+10.$